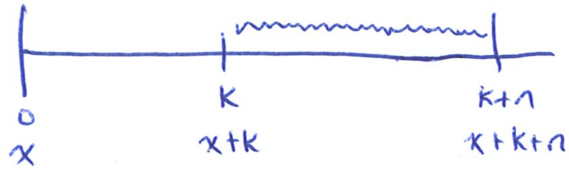
8/29/19

Deferred Mortality (q's)



$Pr((x)$ dies between ages $x+k$ and $x+k+n)$

$= Pr(k < T_x < k+n) \overset{\text{actuarial}}{\underset{\text{notation}}{=}} {}_{k|n}q_x$

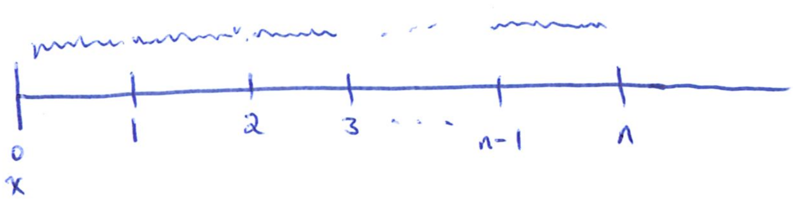
Remarks: 1)

$${}_{k|n}q_x = \begin{cases} {}_{k+n}q_x - {}_kq_x \\ {}_kp_x - {}_{k+n}p_x \\ {}_kp_x \cdot {}_nq_{x+k} \end{cases}$$

$q = 1 - p$

${}_{k+n}p_x = {}_kp_x \cdot {}_np_{x+k}$

2) ${}_nq_x$:



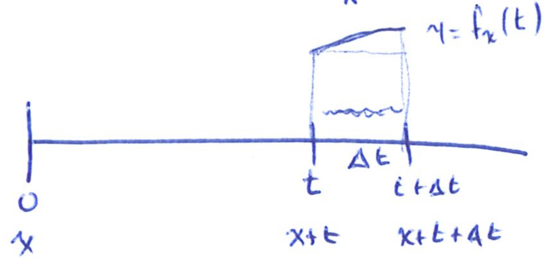
$${}_nq_x = q_x + {}_{1|}q_x + \underbrace{{}_{2|}q_x}_{= {}_{2|1}q_x} + \cdots + {}_{(n-1)|}q_x$$

Example:

$${}_2q_{20} \overset{\text{SULT}}{=} q_{20} + {}_{1|}q_{20} = q_{20} + p_{20} \cdot q_{21}$$

$$= .00025 + (1 - .00025) \cdot (.000253)$$

pdf of T_x : $f_{T_x}(t) = f_x(t)$



$\Pr(\,(x)$ dies between ages $x+t$ and $x+t+\Delta t\,)$

$= \Pr(t < T_x < t+\Delta t) \approx f_x(t)\cdot\Delta t$

<u>Remarks</u>:

1) $_n q_x = \int_0^n f_x(t)\,dt$

2) $_n p_x = \int_n^\infty f_x(t)\,dt$

3) $_{k|n} q_x = \int_k^{k+n} f_x(t)\,dt$

4) $\text{pdf} = (\text{cdf})' \overset{\text{cdf}=1-\text{sf}}{=\!=} -\text{sf}'$

$f_x(t) = {}_t\dot{q}_x = -\,{}_t\dot{p}_x$

$\frac{d}{dt}\left[{}_t q_x\right] = {}_t\dot{q}_x$

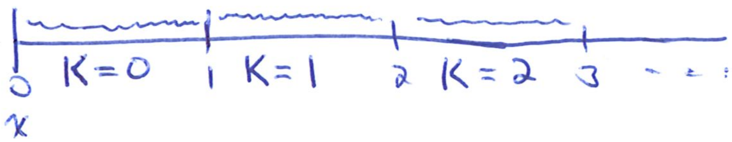
Curtate Future Lifetime Random Variable:

Recall that T_x is a continuous r.v.

Define K_x = integer part of T_x = "floor" of T_x

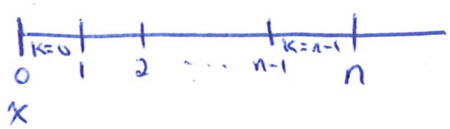
$$K_x = \lfloor T_x \rfloor$$

$\text{supp}(K_x) = \{0, 1, 2, \ldots\} \Rightarrow K_x$ is a discrete r.v.



probability table:

K	Pr
0	q_x
1	$_{1\|}q_x$
2	$_{2\|}q_x$
$\vdots$	$\vdots$



Remarks:

1) $_nq_x = Pr(K < n) \quad \left(= Pr(K \leq n-1) \right)$

2) $_np_x = Pr(K \geq n)$

Other Curtate RV's:

1) $K_x^{(2)} = T_x$, rounded down to lowest $0.5 = \frac{1}{2}$

2) $K_x^{(4)} = T_x$, rounded down to lowest $0.25 = \frac{1}{4}$

3) $K_x^{(12)} = T_x$, rounded down to lowest $\frac{1}{12}$

<u>Example</u>: Suppose $T_x = 12.9$

Then $K_x = 12$

$K_x^{(2)} = 12.5$

$K_x^{(4)} = 12.75$

$K_x^{(12)} = 12.8\overline{3}$

<u>Note</u>: $K_x = \lfloor T_x \rfloor$

For H.W.: $K_x^{(m)} = ?$ in terms of T_x & m